Notes April 20-24, 2020

Happy Monday. Hope everyone is doing well. Hope your distance learning is becoming easier.

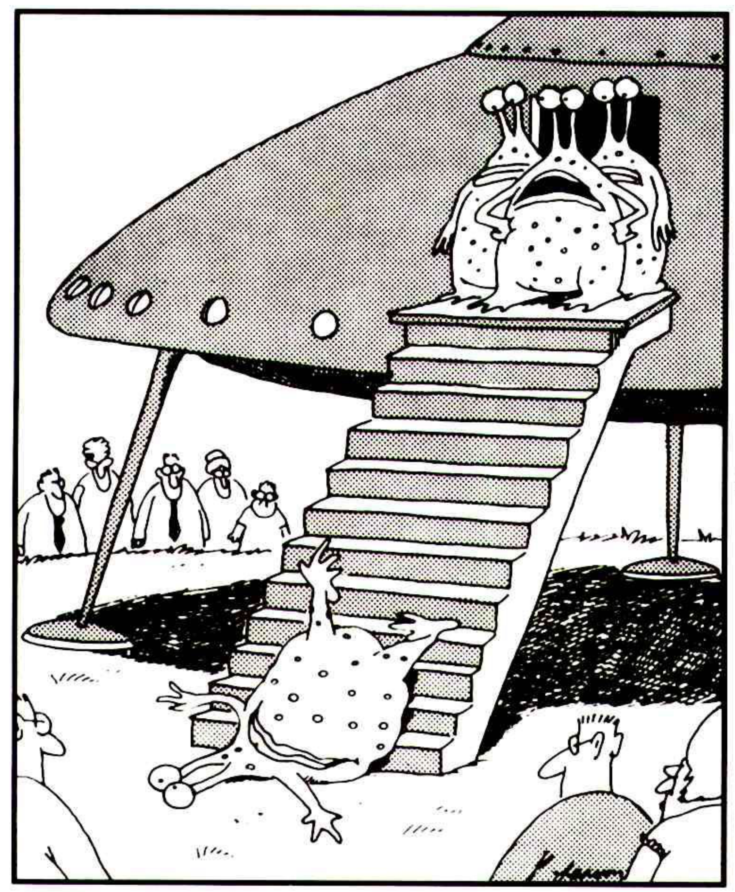
So this week I would like you to complete some lessons on

1. Conservation of Energy

2. Projectile Motion

Two rather big topics so the notes might be a bit long. The assignment will go out on Thursday this week and be due the following Wednesday April 29th, 2020. Be sure to submit the Assignment Work, Power and Universal Gravitation on Wednesday April 22, 2020 this week.

Good luck. Any questions email me.



"Wonderful! Just wonderful! ... So much for instilling them with a sense of awe."

1. Conservation of Energy

We study energy in physics because it allows us another way to determine characteristics about a situation using energy rather than force. A new way to solve the same problems.

Recall: $\mathbf{E_K = mv^2/2}$ and $\mathbf{E_g = mgh}$ from last week.

The Law of Conservation of Energy states that "energy cannot be created or destroyed, only transformed from one type of energy to another". So what does this mean. It means we can't make energy (it just exists) and we can't destroy energy (it can't go out of existence) we can only change it from one form of energy to another form of energy.

For example, if I turn on a battery operated flashlight. By doing this the chemical potential energy stored in the battery is transformed to electrical energy which in turn is transformed to thermal energy in the lightbulb which then turns into radiant energy (light) that we see. The same amount of energy in joules is constant it just changes form.

This law is very powerful because if you can find the total energy in one snapshot of time during a scenario you know the total at all times during the scenario. The only thing that changes is what types of energy you are dealing with.

So we are going to look at a very specific case of conservation of energy where only Ek and Eg exist. So a bit idealized in that there is no friction creating thermal energy and no vibrations creating sound.

So the formulas we will use are

$E_A = E_B = E_c ...$ where E_A is the total energy at position A and E_B is the total energy at position B and E_c is the total energy at position C etc..

What types of energy are present (E_K, E_g or both) at each location are dependent on the situation. Just note that if there is height, h, at a location then E_g exists and if the object is moving at a location with a velocity, v, then E_K exists. The path taken doesn't matter, energy is the same total in all positions.

Example: An inquisitive student throws a 0.5 kg textbook (not a physics one) at 2.0 m/s towards the ground from the top of their balcony, which is 3.2 m high.

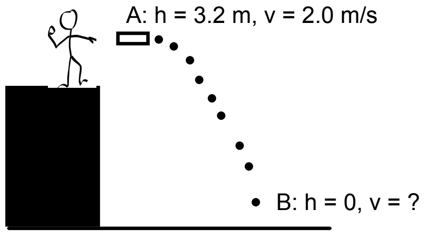
a. What is the E_g and E_K of the book at the beginning of the throw?

b. What the total energy of the book at the beginning of the throw?

c. What is the total energy of the book just before impact with the ground below?

d. With what speed will the book hit the ground?

Let us draw the situation first as diagrams are always useful tools in solving problems. I am going to label the top as position A and the bottom as position B.



a. at position A

$E_g = mgh$

$E_g = (0.5)(9.8)(3.2)$

$E_g = 15.68$ J

$E_K = mv^2/2$

$E_K = (0.5x2^2)/2$

$E_K = 1$ J

b. The total energy at position A, where gravitational and kinetic energy exist is

$E_A = E_K + E_g$

$E_A = 1 + 15.68$

$E_A = 16.68$ J

c. Since there is conservation of energy the total energy at A is equal to the total energy at be. Hence, $E_A = E_B = 16.68$ J

d. But what types of energy make up the energy at position B? Well since the height h is zero (at position B) the gravitational potential energy does not exist here but the object will be moving (just before impact not after) at a velocity v. So kinetic energy does exist at position B. Therefore,

$E_B = E_K$

$E_B = mv^2/2$

$16.68 = 0.5v^2/2$

$2 \times 16.68 = 0.5v^2$

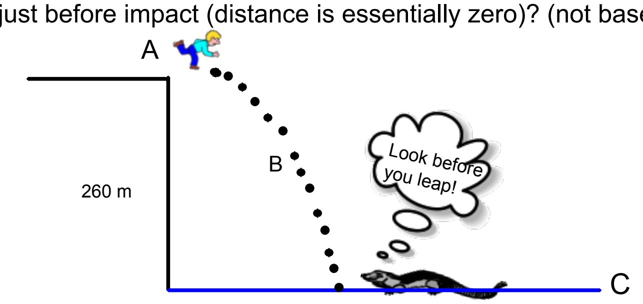
$v^2 = (2 \times 16.68)/0.5$

$v = \sqrt{(66.720}$

v = 8.2 m/s So the textbook hits the ground at 8.2 m/s.

Example 2: Will who is 75 kg runs and jumps from a 260 m high cliff above crocodile infested waters. If he initially starts with a velocity of 2 m/s, what is his velocity

a. at the top (just as he jumps off)?
b. 3/5 the way down?
c. just before impact (distance is essentially zero)? (not based on a true story)



I am going to call the top of the cliff A, 3/5 of the way down B and at the water's surface C.

a. at the top Will has a height of 260 m and a velocity of 2 m/s. So he have both types of energy in the total. However, his velocity is still just 2 m/s.

b. We must always measure the height from the ground up. So 3/5 of the way down is 2/5 of the way up. So h = (2/5)(260) = 104 m from the water. He will also have a velocity that is unknown.

However,

$$E_A = E_B$$
$$mgh + mv^2/2 = mgh + mv^2/2$$
$$(75)(9.8)(260) + (75)(2^2)/2 = (75)(9.8)(104) + 75v^2/2$$

note can only cross out the mass if it is in all terms.

$$2548 + 2 = 1019.2 + v^2/2$$
$$2550 - 1019.2 = v^2/2$$
$$v = \sqrt{(2 \times 1030.8)}$$
$$v = 45 \text{ m/s}$$

c. At position C the height is zero so the gravitational potential does not exist. We can equation position C's total energy to either position A or position B as all are equivalent. I am going to go with A, in case I messed up B's velocity.

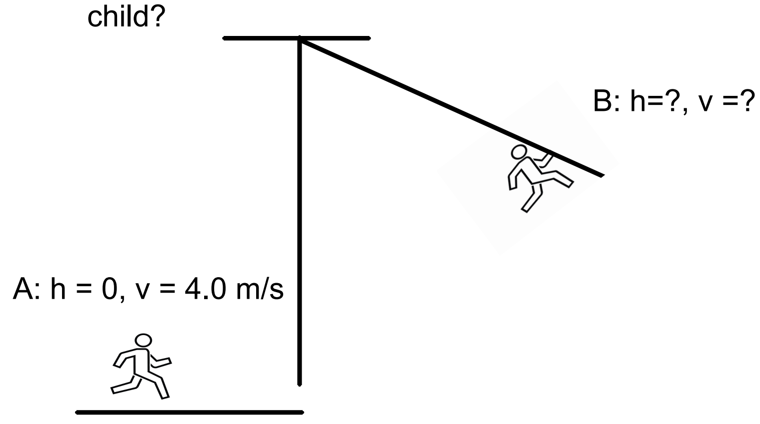
$$EA = EC$$
$$mgh + mv^2/2 = mv^2/2$$
$$(75)(9.8)(260) + (75)(2^2)/2 = 75v^2/2$$
$$2548 + 2 = v^2/2$$
$$2550 = v^2/2$$
$$v = \sqrt{(2 \times 2550)}$$
$$v = 71.4 \text{ m/s}$$

Therefore Will is travelling at very high rate of speed when he hits the water. Crocodiles are the least of his worries.

All this increase in speed comes from the gravitational potential energy of position A changing into kinetic energy as he falls. In real life situations, however, some of the energy will go to thermal energy from the friction with the air, so he probably won't hit at this particular speed. Only in the ideal situation where we ignore air resistance. However, total energy is still conserved with friction involved as well.

Example 3: Several children, pretending that they are playing in the jungle (which is how I broke my arm when I was a kid), suspend a rope from an overhead tree. A child of mass 40 kg running at 4.0 m/s grabs the rope and swings off the level ground.

a. What maximum height does the child reach?
b. How fast would a 25 kg child have to run to reach the same height as the 40 kg child?



a. So we want to equate the total energy at position A to the total energy at position B. But we have no information for position B, or do we? What is the velocity at the top of a swing or a throw. Well for a split second before you start to fall back down the speed is 0 m/s. All gravitational potential energy, no kinetic at position B.

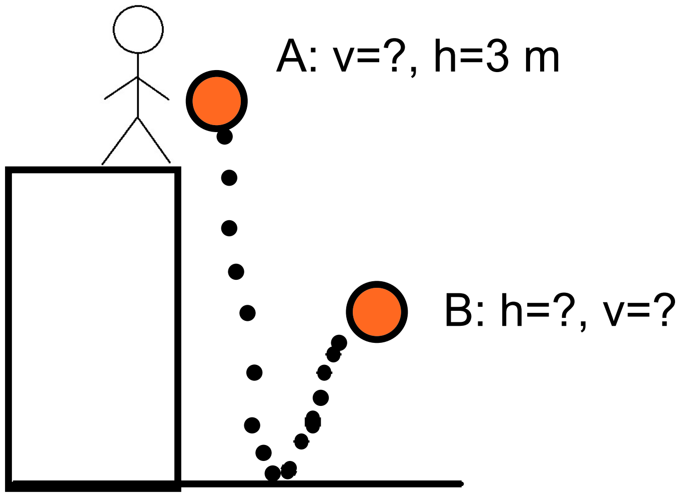
$$E_A = E_B$$
$$mv^2/2 = mgh$$
$$(\cancel{40})(4.0^2)/2 = (\cancel{40})(9.8)h$$
$$8 = 9.8h$$
$$h = 8/9.8$$
$$h = 0.81 \text{ m (not very high)}$$

b. If the 25 kg student wants to reach a height of 0.81 m what speed do they need to run, initially to gain this height.

$$E_A = E_B$$
$$mv^2/2 = mgh$$
$$(25)(v^2)/2 = (25)(9.8)(0.81)$$
$$v^2/2 = 9.8(0.81)$$
$$v = \sqrt{(8x2)}$$
$$v = 4.0 \text{ m/s}$$

The little kid must run at the exact same speed to gain the same height.
Remember, due to gravity all objects, regardless of mass fall and rise at the same rate.

Example 4: Scarlett drops a 1.0 kg bouncy ball from a height of 3 m. If the impact with the floor causes it to lose 15 % of its energy (due to friction, noise etc.) to what height will it bounce?



When an object is "dropped" the initial velocity is zero. So the velocity at position A is v = 0 m/s. When an object reaches the top of a bounce it's velocity again is zero. So the velocity at position B is v = 0 m/s.

When considering questions involving a lose of energy to friction, noise etc. always consider what you get to keep, not what you lose. So if we lose 15 %, that means we get to keep 85 % (100 - 15 = 85) of the energy from position A to position B.

0.85 EA = EB

0.85 mgh = mgh

0.85(1.0)(9.8)(3) = (1.0)(9.8)h

0.85(3) = h

Here I can cross out both mass and g because they are in both terms.

h = 2.55 m   So the ball will bounce 2.55 m off the ground.

Now you try pg 243 # 41, 42, 43, 44

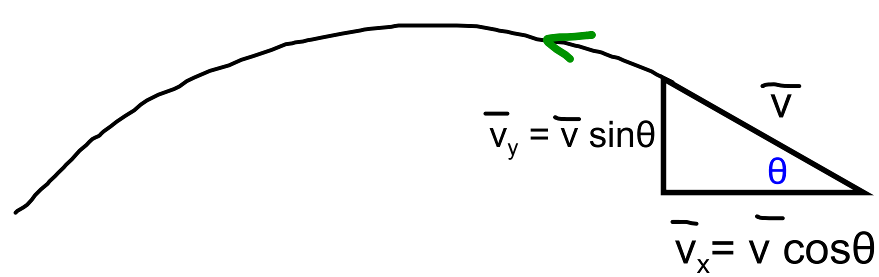
2. <u>Projectile Motion</u>

Now we are going to backtrack and look at a special case of vector motion involving an object that is flung forwards and upwards at the same time. This is known as projectile motion. The thing to note is that the forward and upward motion are independent of one another and only related through time. The projectile spends half the time travelling upward, the half of the time travelling downward and all the time moving forward.

First we need to make some pretty big assumptions so as to not make the question to complicated.

When a projectile is launched from the surface of the Earth it has both horizontal and vertical motion. These motions are independent of each other. The horizontal motion is constant and the vertical motion is accelerated at 9.8 m/s^2 and the time to go forward = the time to go up and down.

Assumptions:
1) Air resistance is negligible.
2) g is constant. The projectile never gets high enough for g to change.
3) the Earth is level. The object lands at the same height as it was launched.
4) The projectile travels in the same vertical plane throughout its flight. There is no side wind pushing the projectile off course.



Here v is the velocity of launch, θ is the angle of the launch, v_x is the horizontal velocity component and v_y is the vertical velocity component.

From these we can build four equations to help us determine some characteristics of the projectile.

1. The maximum range (maximum horizontal distance the object will travel) d_x can be found using

$$d_x = (v\cos\theta)t$$

where t is the time of full flight.

2. The maximum altitude (maximum vertical distance the object will travel) d_y can be found using

$$d_y = (v^2\sin^2\theta)/(2g).$$

Note $\sin^2\theta = (\sin\theta)^2$ on your calculator.

3. The time of half the flight (time to travel upward) is given by

$$t = (v\sin\theta)/g$$

and the time for full flight (time to travel up and down) is given by

$$t = (2v\sin\theta)/g.$$

4. The maximum range d_x can also be found using the formula

$$d_x = (v^2\sin 2\theta)/g$$

This doesn't include time. Only need v and θ to solve. Note $\sin 2\theta = \sin(2\theta)$

Note: the maximum horizontal range always occurs at an angle of 45°.

Example: Miss Takken gets angry and fires off a cannon with an initial velocity of 45 m/s at an angle of 34°.

a) If Brooklyn is standing 195 m away will she be toast or safe? Where will it land in relation to her position?
b) How long does she have to run away?

a. We want to know the horizontal distance that the projectile travels, which we can then compare to 195 m to see if Brooklyn is safe or not! Since we are only given v and θ use equation #4

$$d_x = (v^2 \sin 2\theta)/g$$

$$d_x = (45^2 \sin(2x34))/9.8$$

$$d_x = 45^2 \sin(68))/9.8$$

$$d_x = 191.6 \text{ m}$$

Therefore Brooklyn is safe. It lands 3.4 m in front of her.

b. To figure out how long she has to run away use the time for full flight equation.

$$t = 2v\sin\theta/g$$

$$t = 2(45)\sin(34)/9.8$$

$$t = 5.1 \text{ s}$$

Therefore Brooklyn has 5.4 seconds to run away. However, if she just stood still she'd be fine.

Also, if she does run she should run away from the cannon not towards it.

I guess the bigger questions here are why does Miss Takken have a cannon anyway and what did Brooklyn do? 😄

Now you try pg 114 # 33ab